# Computer-Aided Determination of Microwave Two-Port Noise Parameters

GIUSEPPE CARUSO and MARIO SANNINO

***Abstract*—The least-squares fit of measured noise figures as a function of source admittance is an accurate and rapid method, also convenient from the experimental point of view, to determine linear two-port noise parameters.**

**However, to avoid the erroneous results often obtained by experimenters, this paper presents some criteria to be followed in choosing the proper source admittances.**

**In order to apply the method to microwave two-ports, a relationship relating noise parameters in a linearized form is introduced.**

**The analytical developments are in terms of effective input noise temperature of the two-port.**

**Experimental results for a microwave transistor are also reported as a function of experimental redundancy.**

## I. Introduction

In 1960, the IRE Subcommittee on Noise proposed a manual curve fitting procedure which allows determination of two-port spot noise parameters from the set of experimental data consisting of some values of noise figure measured for different input termination, or source, admittances properly chosen [1].

Since then, two methods of computer-aided determination of noise parameters have been reported in literature.

One of them [2] can be thought of as an automatic version of the graphic procedure suggested by the IRE, which requires tedious and time-consuming adjustment of some input termination admittances with constant real part and of some other with constant imaginary part.

The other one [3] is an application of the least-squares method, which reduces the determination of noise parameters to the solution of a four linear equation system, obtained as fit of noise figures measured in correspondence to source admittances chosen without particular rules. For this reason the latter method is widely preferred to the former.

Unfortunately, this analytical method, although inherently very accurate and also convenient from an experimental viewpoint, can yield erroneous results.

This was interpreted as an apparently high sensitivity to measurement errors and, consequently, application of a more sophisticated fitting routine has been invited.

In the present paper it is shown how these unacceptable results are not inherent in the method, instead, they are due to an improper choice of the input termination admittances when performing measurements. More in particular, if the admittances chosen are in the neighborhoods of some *singular loci* which are recognizable on the Smith chart, large errors may occur because of ill-conditioning of the coefficient matrix of the equation system to be solved.

As proposed here, a way to avoid the above drawbacks is to choose the input termination admittances in the neighborhoods of two or more different singular curves belonging either to the same family or to different families.

To this end, criteria for the realization of the admittances in a quite convenient manner from the experimental point of view are also suggested.

To illustrate the application of the method to the case of microwave linear two-ports, a relationship which relates spot noise parameters in a linearized form is introduced. It differs from the one derived early by Fukui [5] because it involves noise parameters expressed in terms of source reflection coefficients in place of source admittances.

Before concluding, an application of the method to the determination of spot noise parameters of a microwave transistor is reported.

## II. Description of the Method

As well known, the noise behavior of a linear two-port depends on its input termination, or source, admittance $Y_s = G_s + jB_s$ according to the formula [1]

$$F(Y_s) = F_o + R_n \frac{|Y_s - Y_o|^2}{G_s} \tag{1}$$

where $F_o$ is the optimum noise figure, $Y_o = G_o + jB_o$ is the optimum source admittance, namely that value of $Y_s$ at which this minimum noise figure occurs, and $R_n$ noise resistance is a parameter which describes how much the two-port noise behavior deteriorates if the source admittance differs from the optimum one.

We can write (1) in the more useful form [4]

$$F(Y_s) = F_o + N \frac{|Y_s - Y_o|^2}{G_o G_s} \tag{2}$$

where the parameter $N = G_o R_n$ is invariant under transformation of the source admittance due to lossless networks. Therefore, the admittances $Y_s$ can be now measured at the most convenient reference plane located between the

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The authors are with the Universita di Palermo, Istituto di Elettrotecnica ed Elettronica, 90128 Palermo, Italy.

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two-port under test and the noise source, with no effects on $N$ or $F_o$; only $Y_o$ is to be transformed.

However, in representing the noise behavior of microwave two-ports, it is useful to write (2) in terms of the input termination reflection coefficient $\Gamma_s = \rho_s \exp(j\theta_s)$. Introducing the effective input noise temperature $T_e(\Gamma_s)$, (2) then becomes

$$T_e(\Gamma_s) = T_{eo} + 4T_0 N \frac{|\Gamma_s - \Gamma_o|^2}{(1-|\Gamma_s|^2)(1-|\Gamma_o|^2)} \tag{3}$$

where $\Gamma_o = \rho_o \exp(j\theta_o)$ is the optimum value of $\Gamma_s$, and the parameter $T_{eo}$ is related to $F_o$ and standard temperature $T_0$ (usually 290 K) by

$$T_{eo} = T_0(F_o - 1). \tag{4}$$

In order to reduce the least-squares method to the solution of a linear equation system, (3) is transformed in a linearized form. After some algebra we have

$$T_e(\Gamma_s) = a + b\frac{1}{1-\rho_s^2} + c\frac{\rho_s}{1-\rho_s^2}\cos\theta_s + d\frac{\rho_s}{1-\rho_s^2}\sin\theta_s \tag{5}$$

where the *indirect* noise parameters $a$, $b$, $c$, and $d$ are related to $T_{eo}$, $\rho_o$, $\theta_o$, and $N$ by the relationships

$$a = T_{eo} - 4T_0 N \frac{1}{1-\rho_o^2}, \qquad b = 4T_0 N \frac{1+\rho_o^2}{1-\rho_o^2}$$

$$c = -8T_0 N \frac{\rho_o}{1-\rho_o^2}\cos\theta_o, \qquad d = -8T_0 N \frac{\rho_o}{1-\rho_o^2}\sin\theta_o \tag{6}$$

or, conversely

$$T_{eo} = a + \frac{b+\Delta}{2}, \qquad N = \frac{\Delta}{4T_0}$$

$$\rho_o = \left(\frac{b-\Delta}{b+\Delta}\right)^{1/2}, \qquad \theta_o = \tan^{-1}\frac{d}{c} \tag{7}$$

with

$$\Delta = (b^2 - c^2 - d^2)^{1/2}. \tag{8}$$

Let us define the error-function

$$e = \frac{1}{2}\sum_{i=1}^{n}(T_{ei} - T_{esi})^2 = \frac{1}{2}\sum_{i=1}^{n} P_i^2 \tag{9}$$

where $n$ is the number of sets $T_{esi}$, $\rho_{si}$, and $\theta_{si}$ (with $i = 1 \div n$) of experimental data, and $T_{ei}$ represents the corresponding values assumed by $T_e$ given by (5).

Conditions for minimum of error-function are

$$\frac{\partial e}{\partial a} = \sum_{i=1}^{n} P_i = 0$$

$$\frac{\partial e}{\partial b} = \sum_{i=1}^{n} \frac{P_i}{1-\rho_{si}^2} = 0$$

$$\frac{\partial e}{\partial c} = \sum_{i=1}^{n} \frac{P_i}{1-\rho_{si}^2}\rho_{si}\cos\theta_{si} = 0$$

$$\frac{\partial}{\partial d} = \sum_{i=1}^{n} \frac{P_i}{1-\rho_{si}^2}\rho_{si}\sin\theta_{si} = 0 \tag{10}$$

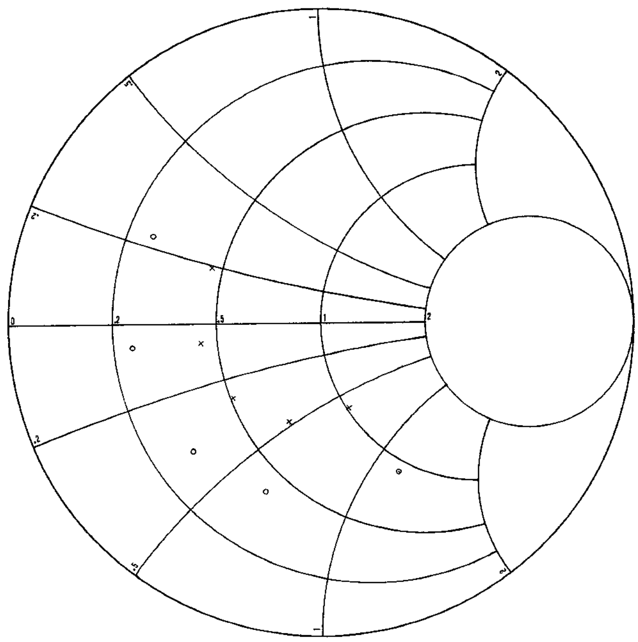

Fig. 1. Source admittances selected to perform the noise temperature measurements. They are obtained by sliding the carriage of a slide-screw tuner for two different penetrations of the tuning slug.

which represent a linear equation system whose coefficient-matrix, through (5) and (9), is given by

$$\begin{array}{lll}
\sum_{i=1}^{n}\frac{1}{1-\rho_{si}^2} & \sum_{i=1}^{n}\frac{1}{(1-\rho_{si}^2)^2} & [kj=jk] \\
\sum_{i=1}^{n}\frac{\cos\theta_{si}}{1-\rho_{si}^2}\rho_{si} & \sum_{i=1}^{n}\frac{\cos\theta_{si}}{(1-\rho_{si}^2)^2}\rho_{si} & \sum_{i=1}^{n}\frac{\cos^2\theta_{si}}{(1-\rho_{si}^2)^2}\rho_{si}^2 \\
\sum_{i=1}^{n}\frac{\sin\theta_{si}}{1-\rho_{si}^2}\rho_{si} & \sum_{i=1}^{n}\frac{\sin\theta_{si}}{(1-\rho_{si}^2)^2}\rho_{si} & \sum_{i=1}^{n}\frac{\sin\theta_{si}}{(1-\rho_{si}^2)^2}\rho_{si}\cos\theta_{si} \\
 & \sum_{i=1}^{n}\frac{\sin^2\theta_{si}}{(1-\rho_{si}^2)^2}\rho_{si}^2. &
\end{array} \tag{11}$$

Through (7) and (8), solution of (10) leads to determination of the noise parameters with an accuracy which increases with the redundancy of experimental data (say, $n = 10$).

On the other hand, by observing the symmetric coefficient-matrix (11) some values $\Gamma_s$ of the source admittance which cause singularity of the matrix can be recognized. For example, singular loci on the Smith chart are represented by those values of $\Gamma_s$ for which the elements of two columns (rows) are proportional; such values are given by

$$\rho_s = \text{const}, \quad \rho_s\cos\theta_s = \text{const}, \quad \rho_s\sin\theta_s = \text{const},$$

$$\tan\theta_s = \text{const}, \quad \frac{\rho_s\sin\theta_s}{1-\rho_s^2} = \text{const} \tag{12}$$

which, therefore, do not define completely the surface represented by (5).

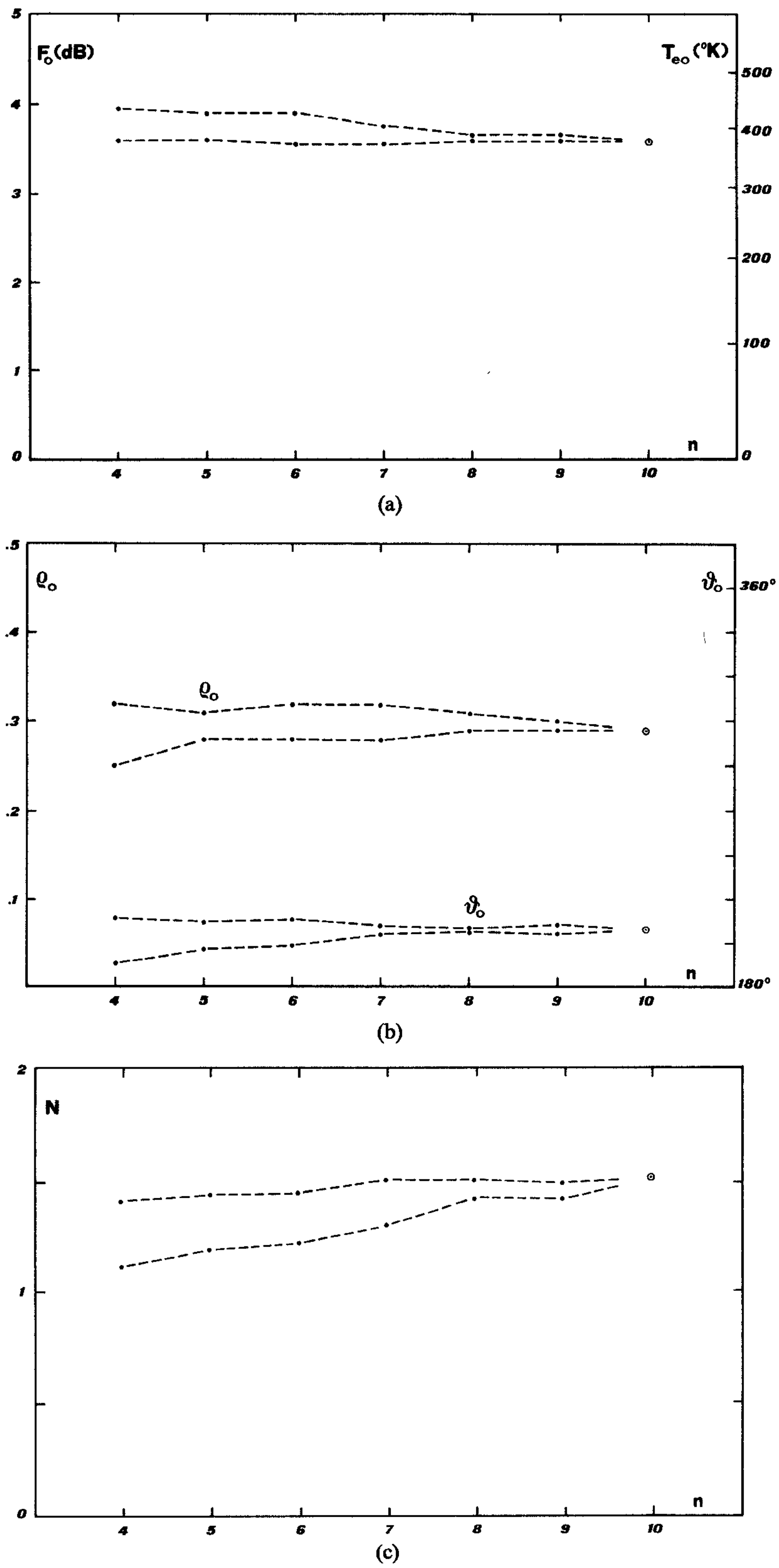


Fig. 2. Noise parameters as functions of redundancy, for a transistor AT-4642, Avantek, at 3.5 GHz ($I_c=5$ mA, $V_{ce}=10$ V). For a given value of redundancy, the dashed curves represent the maximum and minimum values of the noise parameters as the data vary.

More in general, every locus of $\Gamma_s$ which can be seen as a projection on the horizontal plane $(\rho_s, \theta_s)$ of a curve that is the intersection of the surface (5) with other of the same family gives rise to singularity.

Obviously, the values $\Gamma_{si}$ chosen to carry out the measurements will never belong exactly to a singular curve, but if they happen to be in the neighborhood of such a locus, conditions occur for ill conditioning of the matrix and, consequently, unacceptable errors in the parameter determination may follow. This has been proven by many tests which have also shown that under these conditions the results obtained vary strongly as the number of experimental data varies. In addition, in most cases, they are without physical meaning. In order to avoid the above inconvenience, input termination admittances are to be chosen properly in performing measurements. A way of doing this is to choose the admittances in the neighborhoods of two different singular curves, because this will completely define the surface given by (5). A practical suggestion to easily apply the above concept is to use a (coaxial or waveguide) slide-screw tuner as source admittance transformer, which permits the realization of admittances defined by $\rho_s = \text{const} = C_1$ and $\rho_s = \text{const} = C_2$ simply by sliding the carriage for two different penetrations of the tuning slug.

## III. Experimental Verifications

In order to show the validity of the method presented here for computer-aided determination of noise parameters, some experimental results obtained for a microwave transistor (Avantek-AT 4642; $I_c = 5$ mA, $V_{ce} = 10$ V) at 3.5 GHz are reported in this section.

As admittance transformer network a coaxial slide-screw tuner has been employed, and a set of admittance values has been obtained, as above suggested, by sliding the carriage for two different penetrations of the tuning slug. Obviously, admittances corresponding to low values of measured noise temperatures have been selected to improve the accuracy. In Fig. 1 the source admittances are represented on the Smith chart.

Effective input noise temperatures have been then measured and the noise parameters derived by processing the experimental data through a well-known computing program for linear equation system solution.[1]

The following results have been obtained: $T_{eo} = 377$ K ($F_o = 3.6$ dB), $\rho_o = 0.3$, $\theta_o = 197°$, $N = 1.53$ ($R_n = 43.5$ ohm)[2].

The noise parameters have been also computed by processing a number of experimental data lower than 10, provided that, as previously stated, data are selected in correspondence to admittances some of which belong to the inner locus of Fig. 1 and some other on the outer one.

The values of the noise parameters so obtained are reported in Fig. 2 as functions of redundancy, i.e., of the number of the data processed.

From this figure it appears that at a given value of redundancy, slight variations of noise parameters occur as the data vary, provided that the redundancy is sufficiently large (say $\geqslant 7$).

The experimental verifications have been carried-out by means of a measuring system ($Y$-factor method) equipped with variable attenuator, test receiver, and gas-discharge noise source (excess noise ratio: $15.6 \pm 0.25$ dB).

## IV. Conclusion

An application of the least-squares method to the determination of microwave two-port noise parameters is presented, and it is shown how the erroneous results often obtained by experimenters can be avoided by properly choosing two port input termination admittances in performing the measurements. To this end, some criteria to follow are given, together with suggestions for their practical application.

Some experimental results for the case of microwave transistor noise parameter determination are also reported.

## Acknowledgment

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[1]IBM subroutine SIM Q, translated into BASIC language for an HP 9830A desk computer.

[2]The measured optimum noise figure is close to the typical value of 3.3 dB given by the transistor manufacturer.

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